THE TRANSFORMATION PROBLEM.
The Maths behind the Science.

In my previous postings on the transformation problem, as expounded by Marx in Chapter 9 of Volume 3, the reader can find the two variables that need to be solved. In the aforementioned Chapter Marx did not extend his example by repricing the five capitals under investigation. Nor did he split the 26 of surplus value that needed to be redistributed into two streams: one to reprice capital and the second to adjust the profit made necessary by the repricing of this capital. Without so doing the transformation solution is impossible. By taking these factors into consideration the following holds true: the equalisation of the rate of profit does not take place based on the old market values, but on the newly repriced capital. This posting adds the third and final variable needed to model the "supply side" of the economy; the effect on prices of production resulting from the changes to proportion of circulating capital relative to the unconsumed stock of capital.

In my view the reason that all the recent solutions to the transformation problem have failed is their failure to reprice the capitals which give rise to the market prices in circulation. Once this is done, input prices derived from these repriced capitals are in fact measured in prices and not values. This extension was never considered by Marx and it was missed by all the "Marxists" who followed him. Secondly a failure to reprice capital obscures the fact that once this is done, individual profits need to be adjusted otherwise an average rate of profit itself is impossible. Finally, once it is recognised that profits need to be adjusted subsequent to the repricing of capital it becomes clear that the redistribution of surplus value has to be split into two streams.

It is important to stress at the outset that the original values are treated as market values and not individual values. Individual values cannot be transformed into prices, only into market value. Market value, unlike abstract value is concrete. Abstract value is a simple average devoid of difference, and, is used purely for the purpose of investigation and presentation. Concrete value represents weighted averages which accounts for differences, and not only differences, but the actual weight of the differences. Thus, the market value of a product is the weighted average labour time expended in its production, because only the weighted average labour time when multiplied by the number of units produced can equal the total labour time expended on that product.

The capitals considered here either coincide with the market value for that industry or can be considered whole industries.

Two examples will be analysed comprising three capitals. We are unconcerned whether or not each capital represents a department of production. However, it is convenient to assume that Capital (2) comprises articles of consumption destined for workers, because being average their prices do not deviate from values leaving the price or value of labour power unaltered by the redistribution of surplus value.

In the first example we will examine what happens when all the constant capital is used up in the cycle of production, and, in the second more complex example, when only a portion of that capital is consumed within the period of production. The second example conforms to Marx's stricture at the beginning of Chapter 9 in Volume 3 that in real life not all the constant capital is used up, and that proportionately, less is used up in the higher composition capitals. This relative consumption is the basis for deriving the third variable which forms the objective of this posting.

Finally, as with Marx, simple reproduction in a closed system is assumed. In all cases monetary demand remains unaltered by changes in production being fixed at 320, 300 and 280 respectively for the output of the three capitals. This posting dispenses with Marx's tables in order to simplify the maths behind the transformation of market values into market prices of production. The original posting which employed the tables used in Chapter 9 can be found on this website by following this link: https://theplanningmotivedotcom.files.wordpress.com/2015/09/transformation-solution-pdf.pdf

Case 1.

Here three capitals are chosen, and in all cases the entire capital is completely consumed within the period of production and circulation.

Table 1.

	c	+	v	+	s	=	market value
(1)	80		120		120		320
(2)	100		100		100		300
(3)	120		80		80		280
Totals	**300**	**+**	**300 (= 600)**	**+**	**300**	**=**	**900**

The three capitals differ in composition, but in each capital the rate of exploitation is the same at 100% (s/v). Although each capital adds up to 200 (c + v) yielding a total of 600, the ratios between c and v vary. In capital (1) the value composition of capital ($^{c}/_{v}$) is 67% or 80/120 for (2) it is 100% and for (3) it is 150%. Capital (3) therefore has the highest composition and capital (1) the lowest. In Capital (3) each worker works with more means of production than is the case in Capital (1). We note that in all cases, Capital (2) is the average.

In order to determine the rate of profit, the table can be structured to reflect the amount of capital compared to the surplus value it produces.

Table 2.

	c	+	v	=	capital	surplus	rate of profit
(1)	80		120		200	120	60% ($^{120}/_{200}$)
(2)	100		100		200	100	50% ($^{100}/_{200}$)
(3)	120		80		200	80	40% ($^{80}/_{200}$)
Totals	300		300		600	300	50% ($^{300}/_{600}$)

We note that due to their different compositions each of the three capitals has a unique rate of profit. Capital (3) with the highest composition has the lowest rate of profit and Capital (1) with the lowest composition has the highest rate of profit. The average rate of profit of 50% coincides with the rate of profit for Capital (2) the average capital.

Clearly three rates of profit cannot co-exist between industries just as different prices for the same commodity cannot exist within an industry. In real life the movement of capital between industries tends to erode differences resulting in an average rate of profit which in this case is 50%. The question is posed, how much value, and, in what direction must it be transferred, to yield an average rate of profit? The answer is 20 which must be moved from (1) to (3) after which all three capitals enjoy the same rate of profit of 50%. Following this movement, Table 1 now appears thus:

Table 3. (Ideal Stage)

	c	+	v	+	s	=	price of commodities	rate of profit
(1)	80		120		100		$300 (320)	50%
(2)	100		100		100		$300 (300)	50%
(3)	120		80		100		$300 (280)	50%
Totals	**300**		**300**		**300**		**$900**	**50% average**

We note that only the s column has changed. Instead of selling at 320, the output of capital (1) now sells for only 300. Conversely, capital (3) now sells its output at 300 instead of 280. Exchange previously equal has now become unequal. Capital (1) loses out because it receives only $300 in money in exchange for 320 in value. On the other hand, Capital (3) gains because it receives $300 in money compared to 280 in value. Capital (1) is poorer and Capital (3) is richer. The inequality favours (3).

I have added in the $ sign to show that output is now priced, and I have changed the heading over column 4 from market value to prices of commodities which must not to be confused with market price because the former are ideal prices while market prices are real. If we assume that the total number of commodities produced is 900 and the market value of each is $1, then total market value and total prices would remain at $900. However, what has transpired in Table 4 because of unequal exchange, is that not all the commodities sell for $1 as shown in the table below.

Table 4.

	Market Value	Market Price	Loss or gain	Unit price
(1)	320 (320 units)	300	-20	93.8 cents
(2)	300 (300 units)	300	0	$1.00
(3)	280 (280)	300	+20	$1.071
	900	**900**	**0**	**$1 average (rounded)**

Table 3 was an ideal situation. In reality competition is not based on maths. It is based on physical movements. When industries yield different rates of profit, capital exits the industries with lower rates of profit and enters industries with higher rates of profit. The industries from which capital exits, tends to see profits rising due to declining supply, whereas profits tend to fall in the industries attracting investment as supply increases there.

It is possible to model the change in the physical balance of production to equalise the rate of profit. Here two assumptions shape our modelling. The monetary demand is unaffected, it remains at 320, 300 and 280 respectively. Secondly the capital moves across from (3) to (1) in a manner that is consistent with the composition found in (1), in order to leave the overall composition of capital unaltered.

How much capital needs to be transferred? This simple question has flummoxed most theoreticians. It is tempting to say that $20 worth of capital moves from (3) reducing it to 260 and increasing the capital in (1) to $340. If we were to do this we would be quite wrong. We recall in Tables 1 & 2 that the social product is 900 while the capital employed is only 600. This 600 is reinvested while the profit of 300 is withdrawn to be unproductively consumed by the capitalists. Only the value returned back to production, in this case 600 can be the vehicle for price changes.

600 out of 900 is two-thirds. It thus follows that instead of 20 moving between capitals only two thirds can or 13.3, not 20. The balance of 6.7 is needed to re-adjust the mass of profits in order to yield a rate of profit of 50% on the repriced capitals. If the 20 is not split in the ratio of capital to social product,

the repricing of capital is either over or understated, and secondly, there are insufficient profits to yield an average of 50% on all three capitals

If we were to restate this relation in aliquot shares, then the equivalent change between capitals (1) and (3) is 6.7% ($^{13.4/}$200). Now it is not important whether this movement of capital is incremental (the iteration referred to by Shaikh) or whether it is done in one fell swoop. The importance is to locate the end result, the change in market value that corresponds to the prices found in Table 4.

Two tests will be applied to confirm the accuracy of the changes. Firstly, do the new prices equalise the rate of profit? Secondly is there a shift in the mass of profits of $6.7 in opposite directions in accordance with the repriced capital using Table 3 as the template.

This movement is shown in Table 5 below.

Table 5. (Intermediate Stage)

	c	+	v	+	s	=	market value	rate of profit	(*adjustment*)
(1)	88		125.4		125.4		339	59%	*(+13.4)*
(2)	100		100		100		300	50%	*(0)*
(3)	112		74.6		74.6		261	40%	*(-13.4)*
Totals	**300**		**300**		**300**		**900**	**50% average**	

We note no change in total value, in composition and in individual rates of profit. However, the volume of production has changed. In the case of (1) production has increased from the original 320 units to 339 and in the case of (3) it has shrunk from 280 to 261. This shift of 19 approximates the necessary shift of value amounting to 20 identified in Table 3 (In Case 2, I refine the maths to yield 20). The first test are the prices, do they approximate the modelling done earlier of 94 cents, $1 and $1.07. In the case of Capital (1) output of 320 units has increased to 339 yielding a unit price of 94 cents ($^{320/}{}_{339}$), and in the case of capital (3) its selling price is now $1.07 ($^{280/}{}_{261}$).

This is close enough to our original modelling to merit no further consideration. And when money equivalents of $320, $300 and $280 are spent on the output of (1), (2) and (3) respectively, then each achieves a rate of profit of 50% because of the changed price per unit.

The final consideration is whether the mass of profits deviate by the required $6.7 in both directions. They do. In (1) profit rises from 100 (Table 3 - the ideal stage) to $106.6 (the real stage) and in the case of (3) it falls to 93.4 from 100 (Table 3). Thus, the two tests have been met which means the prices of $0.94. $1.00 and $1.07 are the market prices that arise from the movement of capital and the subsequent shift in production. And they are correct because 13.4 of the original 20 surplus redistributed has repriced capital, and 6.6 has adjusted profit. (Note: for this example, total cost price is equal to total capital invested because all the capital is consumed in each cycle of production.

Table 6 (Final Stage = prices of production)

	c	+	v	+	s	=	market price	rate of profit
(1)	88		125.4		(125.4 – 19)		320	50%
(2)	100		100		100		300	50%
(3)	112		74.6		(74.6 + 19)		280	50%
Totals	**300**		**300**		**300**		**900**	**50% average**

Or conversely:

Table 7. (Final Stage = prices of production)

	Market price	less	(cost price) (or capital)	=	adjusted profit	rate of profit
(1)	320		(88 + 125.4)		106.6	50%
(2)	300		(100 + 100)		100	50%
(3)	280		(74.6 + 112.0)		93.4	50%
Totals	**900**	**less**	**600**	**=**	**300**	**50% average**

Of course, if we had used the full 20 to appreciate or depreciate capitals (1) and (3), or what is the same thing reprice them, the result would have been an error. Let us examine the effect on Capital (1). Instead of c being priced at 88 and v at 125.4 they would have been priced at 91.9 and 128.1. Assuming the same rate of exploitation in (1) c + v + s would have been 91.9 + 128.1 + 128.1 = 348.1, yielding a difference far removed from 320, namely 28.1 and not around 20. In any case, physically it would have been impossible to shift all the 20 because 6.6 would have been consumed unproductively in the form of profit rather than circulating (redistributed) as capital.

If I was to identify the two breakthroughs in my original article it was the identification that the determination of aliquot changes to capital was arrived at by dividing the amount of surplus value that had to be redistributed, not over the capital, but the social product which includes profit (s). The splitting of surplus value into two streams, the first to reprice capital and the second to adjust profits allowed the following to hold true; namely that the rate of profit was now based, not on the original value of the capital, but on the capital as it was now repriced. Encapsulated in these two identifiers is the solution to the transformation problem. However, as Case 2 will reveal, there is still a third variable that needs to be accounted for.

Thus, the market value found in Table 5 resulting from the physical change in production would give rise to the market prices found in Table 6. We therefore find in Case 1, the simplest exposition, that the mathematical modelling can be replicated by the actual movement of capital. Market value and the composition of capital interact. Variations in composition dictate the amount and direction surplus value needs to be redistributed, and therefore it provides the hypothetical limits to the direction and movement of capital to achieve the same result.

Case 2.

We will continue to use the three capitals above. Only now c is not consumed at a rate of 100%. This is in accordance with Marx's injunction in the opening part of Chapter 9 that it is unlikely for all the constant capital to be consumed, and that generally, a smaller proportion of the capital is consumed in higher composition capitals.

I have taken these assumptions and modified the examples accordingly. In the case of (1) instead of all 80c being consumed only 60c is consumed, leaving 20c unconsumed. This amounts to 75% of c in (1) consumed, whereas in 3 only 33% is used up or 40c out of the 120c.

Table 8.

Value of c	Original c	c used up +	v +	s =	market value	c left over
(1)	80	60	120	120	300	20
(2)	100	50	100	100	250	50
(3)	120	40	80	80	200	80
Totals	**300**	**150**	**300**	**300**	**750**	**+ 150 = 900 total value**

The first thing to note is that only 150 of the 300 constant capital (means of production) is invested in production. 150 is left over or unconsumed (highlighted box). The result is that the social product previously totalling 900 is reduced to only 750. Similarly, the capital thrown back into production, previously 600, is now reduced to only 450 (150c + 300v).

However, this does not affect the rate of profit which is calculated on the entire capital of 600 and not simply on its circulating part amounting to 450. Only now, a redistribution previously carried out by circulating capital of 600, has to be accomplished by only 450. This means a greater deviation of prices from values than before so that 450 can achieve what 600 achieved before. Further, whereas earlier the distribution was proportionate between (3) and (1), now it cannot be because of the disproportionate consumption of capital which amounts to 75% in (3) but only 33.3% in (1).

We can ignore Capital (2) because it remains the average capital despite half of its capital being consumed. The price redistribution occurs only between (3) and (1).

To achieve the same rate of profit still requires a movement of 13.4 in capital and a 6.6 in profit as before. What will have changed is the prices of production needed to accomplish this. Previously a deviation of 6.7% or $^{13.4/}{}_{200}$ sufficed. Now it is $^{13.4/}{}_{140}$ in the case of (1) and $^{13.4}/_{120}$ in the case of (3). In terms of aliquot shares this represents a shift of 9.6% and 11.2% respectively. Thus, the shift in prices must be based on these shares. Put another way, prices will deviate from market values by a greater degree because aliquot shares of 9.6% and 11.2% are bigger than 6.7%.

Accordingly, the price of each item produced by (1) will fall from $1.00 to 90.4 cents (100% - 9.6%) due to the rise in the volume of production. Conversely the price of each item produced by (3) will rise to $1.12 due to the fall in production. All this presupposes the prior movement in capital from (3) to (1) causing the change in volumes. This compares to the smaller change of 94 cents and $1.07 when all the capital was consumed.

When multiplied by the capital in circulation valued originally at 140 and 120 respectively the following results are found.

Table 10a (Price of Commodity).

	Total + Value of c	Variable + capital	Surplus = value	Modified Market Value
(1)	80 x 1.096 = **89.4**	120 x 1.096 = **129**	**129**	**347.4**
(2)	100	100	100	300
(3)	120 x .904 = **110.5**	80 x .904 = **71**	**71**	**252.5**
Totals	**300**	**300**	**300**	**900**

Table 10b. (Market Price)

	Capital +	profit =	market price	rate of profit
(1)	89.4 + 129 = 218.4	(129 -20) =109	327.4	50%
(2)	100 + 100 = 200	100	300	50%
(3)	110.5 =71 = 181.5	(71 +20) = 91	272.5	50%
	600	300	900	50%

(Note 1.)

Thus, a market value of 347.4 for capital (1) corresponds to a market price or market price of production of $327.4 and a market value of 272.5 for (3) corresponds to a market price of $252.5. To complete our investigation, it is necessary to pursue a final example. We have already detected that market prices deviate from market values depending on the value of circulating capital as a share of total capital. From this observation the following law arises. The smaller the share of circulating capital the greater must be the deviation of market prices from market values and the greater the share of circulating capital the smaller must be the deviation.

In the final example the share of circulating capital in (1), (2) and (3) has been reduced by 10. As a result, the value of capital in circulation falls from 450 to 420. Accordingly, the aliquot shares change as well. In the case of (1) where the capital in circulation falls from 140 to 130, its aliquot share rises from 9.6 to $^{13.4}/_{130}$ or 10.3%, while in the case of (3) it rises 12.2% ($^{13.4}/_{110}$)

Table 11a (Price of Commodities).

	Total + Value of c	Variable + capital	Surplus = value	Modified market value
(1)	80 x 1.103= **91.1**	120 x 1.103 =**130.7**	**130.7**	**352.5**
(2)	100	100	100	300
(3)	120 x .878 = **108.9**	80 x .878= **69.3**	**69.3**	247.5
Totals	**300**	**300**	**300**	**900**

Table 11b (Market Prices)

	Capital	profit	price of production	rate of profit
(1)	91.1 + 130.7 = 221.8	(130.7 -20) = 110.7	332.5	50%
(2)	100 + 100 = 200	100	300	50%
(3)	108.9 + 69.3 = 178.2	(69.3 +20) = 89.3	267.5	50%
	600	**300**	**900**	

Now it is the case that the market price for (1) has risen from $327.4 in Table 10 to $332.5 and, it has fallen from $272.5 to $267.5 for (3). This is the anticipated result. It is important to note, these greater variation in prices are compensatory and thus total prices and values continue to correspond to 900.

Both market prices however are 20 distant from their market value (Table 11a) with (1) being below and (3) being above in conformity with Marx's method.

Marx's deep insight into the nature of capitalism.

> *Yet in order not to arrive at totally incorrect conclusions, we must not take all the cost prices as 100." (Karl Marx, Chapter 9, Volume 3, page 255 Penguin Edition.)*

It is now possible to model the entire supply side of the economy, a process which begins with aggregate capital (and its division into stock and circulating), aggregate rates of surplus value and average rates of profit in order to establish the base line for all the calculations based on industry specific deviations from these aggregates. The three variables that will enable us to reconcile concrete prices with underlying values are as follows. Firstly, the differences in the value compositions of capitals with appropriate weightings, establishes how much surplus value needs to be redistributed between industries. The rate of surplus value determines the proportions going to reprice capital and the proportion needed to adjust profits to achieve an average rate of profit. Finally, the third variable, the proportion of capital that is in circulation compared to the stock of capital is then taken into account to prepare the final adjustment of market prices..

With reference to the third variable, the smaller the proportion of circulating capital the greater must be the deviation of market prices of production from underlying market values, and the larger the proportion, the smaller the deviation. Only now can we see the importance of Marx's injunction that any modelling of the capitalist economy requires as one of its assumptions, that only a fraction of constant capital is consumed within a specific cycle of production. While Marx warned against any method which utilised a 100% consumption of constant capital per period of production, he never explained its importance. That importance is now established. Without including the third variable, we are unable to calculate with any precision the final deviation of prices and values. Of course, it goes without saying that the stock of constant capital includes the fixed element which includes structures, machinery, equipment, computers and so on.

The move to an objective pricing system from an indirect pricing system (law of value).

Finally, while these three variables allows for the derivation of a mathematical formula which can be applied to the economy, this is regrettably beyond my mathematical abilities.

The importance of developing a methodology to translate market values into market prices of production takes two forms, the first ideological and the second practical. The ideological form is the ability to defend the law of value as developed by Marx. It is all very well to repeat the law by rote, but this will not suffice. We need to be able to explain concretely how prices emerge from values. The solution to the transformation problem needs to be bullet proof.

Nor is it sufficient to endlessly criticise capitalist economists for being intoxicated by surface appearances, the hallmark of any vulgar discipline. The marginal theory of utility which passes for an explanation of price has more in common with the psychiatric profession than with the economic profession. And as with all vulgar observers dazzled by surface reflections, they do not seek to answer the more fundamental questions as to why the phenomenon exists in the first place, why it emerges where it does, why it changes and of course why the surface is rippled by contradictory forces.

Further, those who worship at the alter of marginal utility, cannot explain the emergence of mass markets. Mass markets are the product of changing cost, not utility. Luxury goods become common, not because their utility falls but because their cost does. Henry Ford's quadrupling of productivity by means of bringing the product to the worker rather than the worker to the product - the assembly line - cheapened cars and transformed cities. The same goes for white goods in the 1930s, jet travel in the 1960, electronics in the 1980s, and flat screen TV's and Smartphones in the noughties. In all cases utility rose but costs fell. The consumer society owed everything to modern production methods which in all cases advanced labour productivity.

Thus, all roads lead back to the cost of production and its sources. In their hidden and unguarded moments, the capitalists let the cat out of the bag. The wealth of a nation they declare can be reduced to demographics multiplied by productivity. The more workers there are available for production and the greater their productivity, the greater the potential national wealth. These days they are most anguished because not only are populations stagnating or declining as in Japan, but productivity is flatlining. Where oh where are their profits to come from.

Further, the capitalist's approach to price is schizophrenic. On the one side, in their grand and not so grand institutions of learning they teach bright young things about marginal utility. On the other side, their accountants and statisticians dispense with use value (utility) to deal with the commonality of all commodities, their exchange values. The financial accounts of any corporation and the national accounts owe everything to Marx. The division of these accounts into the "Trading (Manufacturing) Account" and the "Profit and Loss Account" mirrors Marx's division of labour into productive and unproductive, or what is the same thing, labour that adds gross profit and labour that reduces it. Similarly, the national accounts which are based on the value added by final sales, a methodology first described by Marx.

Psychiatrists like saying that the unconscious rules the conscious mind. This certainly applies to capitalism. It appears that the unconscious processes in the capitalist economy, the daily ritual of costing production, truly rules the conscious mind, the obsession with use values, an obsession which if left unattended, would quickly lead to economic madness.

In sum, being able to solve the transformation problem is of ideological merit in the period prior to the revolution. It shows we are able to substantiate the labour theory of value enabling us to explain paid and unpaid labour (exploitation), and, additionally how in a period of economic crisis the capitalists end up squandering this unpaid labour and are therefore underserving and unworthy of it.

It is only in the period after the revolution and the abolition of capitalist private property, that this transformation solution becomes practical. Capitalist prices need to be unravelled as we move to a pricing system based on average weighted labour times. By being able to understand and estimate the deviations of prices from values (or in this case actual costs of production) informed decisions of what and how to produce products can be made. What appeared to be low-cost may be higher-cost and vice versa. Out of the confusion order can be detected.

Knowing what things actually cost before they can be measured directly brings forward the day when conscious planning can be undertaken. The formula here described makes that process faster and clearer. The juvenile Marxists talk of socialism as the ending of prices, a priceless Nirvana. They are clueless as to how complex the process of moving from prices which rewards profits to prices which rewards labour truly is.

Note 1, as the raw multiplication yields a discrepancy of 2%, I have done a final adjustment to arrive at repriced capitals of $218 and $181.5 respectively. This in no way detracts from the methodology but makes the relationships clearer. All that has been done is the following: (1) 80 x 1.096 = 87.7 and (3) 120 x .904 = 108.5 Together they add up to 196.5 which when compared to the original 200 yields a difference of 1.8%. Applying this 1.8% difference yields the figures for c + v found in Tables 10a and 10b.)

Brian Green, April 2019