THE PRICE OF PRODUCTION IS THE PRICE WHICH YIELDS AN AVERAGE RATE OF PROFIT, NOT ON THE OLD CAPITAL, BUT THE NEWLY PRICED CAPITAL.

Establishing the connection between value and price is a task of the highest order. Without it there can be no confirmation of the law of value, no explanation of the nature of exploitation, and no accounting for the general movement of prices in a capitalist economy. This article does not seek to explore the history of solutions to the "Transformation Problem". Rather it is concerned with a novel approach to this problem which the author considers to be in harmony with Marx's assumptions found in Chapter 9 of Volume 3. (All references belong to the Penguin Edition of Capital, printed in 1991).

Multiple Transformations.

There are multiple price transformations, not only one, caused by the deviation of prices from values. Only the three major general deviations will be examined here. More specific deviations, that is momentary ones, such as international terms of trade, or the use of low tax jurisdictions to manipulate invoiced prices, are ignored. The three major or general sets of deviations are listed below.

1. The transformation of individual values into market value and hence market price within an industry.
2. The transformation of market prices into market prices of production to achieve an average rate of profit between industries, whose technical requirements result in capitals of differing composition.
3. The industrial cycle which ensures that in the down phase prices trend below values compared to the up phase where they trend above values.

Market value. Before proceeding it is important to point out that the deviation of prices from values always implies unequal exchange. In Volumes 1 and 2, Marx's investigation of the capitalist social relation requires a degree of abstraction so that no incongruity exists between value and price. In short, Marx assumes exchanges to be equal because capital and labour power have all been rendered similar. Similar means a simple average where all differences have been removed (abstracted out).

Only in Chapter 10 of Volume 3, does Marx move from his treatment of labour as abstract to more concrete (complex) forms and therefore from simple value to market value, reintroducing the elements that make value historical. Marx replaces average labour times with weighted average labour times. He recognises that when competition erodes the differences in a locality for the same product, yielding the market price, that market price cannot be based either on a simple average nor on individual values. In real life, individual producers are not average; they not only differ in their productivity, but also in the volume (weight) of their production.

Market value based on weighted averages is best described via the format of a table.

Table 1.

company	Individual cost of production	Volume of production	Assumed Value of individual sales	Simple average	Weighted average
(1)	(2)	(3)	(4)	(5)	(6)
A	120	80	9,600		
B	100	100	10,000		
C	80	160	12,800		
TOTALS		**340**	**32,400**		
AVERAGES	**120+100+80/3=100**		**32,400/340=95**	**100**	**95**

The table describes an industry with three producers. Each producer is distinguished by their own individual cost of production and their own unique volume of production. They are all different. Company A is the high-cost producer, B is the average cost producer and C is the low-cost producer. The average cost of production is 100. Additionally, they produce different volumes, in short, the weight of production differs between them.

If individual prices, instead of market prices prevailed, namely 80, 100 and 120, then the sales produced by (A) would yield 9,600, (B) would yield 10,000 and (C) 11,200. Clearly competition would prevent this from happening because three different prices in the locality for the same item could not co-exist. (A) would be foolish not to reduce its selling price to avoid losing sales, and, (C) would be wise to raise its price to take advantage of a higher price. The question is posed, will competition result in a market price of 100 based on the simple average cost of production, or 95, the weighted average cost of production which also accounts for the volume of production.

The answer is 95 and not 100. The reason: only the weighted average cost of production of an item when multiplied by its volume can yield the total labour time expended producing it. The simple average will not do so, except where it coincides with the weighted average. In the case of the simple average above, the result will be 100 x 340 (total items) = 34,000 which overstates the amount of labour time expended. Marx was quite clear in Chapter 10 that under normal market conditions, when an industry is dominated by a "preponderance" of below average producers, the market value:price sits below the average and vice versa when the preponderance of production sits above the average.

In the field of science, it is recognised that the method of enquiry and the presentation of results are complimentary. Therefore, it is difficult to understand why, in editing Volume 3, Friedrich Engels decided to locate the chapter on market value (Chapter 10) after the chapter dealing with market prices of production (Chapter 9). As Marx observes*: "What competition brings about, first of all in one sphere, is the establishment of a uniform market value and market price out of the various individual values of commodities. But it is only the competition of capitals in different spheres that brings forth the production price that equalises the rate of profit between those spheres. The latter requires a higher development of the capitalist mode of production than the former."* (page 281)

Throughout Chapter 10 Marx refers to prices of production displacing market prices as capitalism develops, and by this we infer, as credit markets evolve and capital is socialised, improving the fluidity of capital between industries. Thus, logically in terms of historical presentation, Chapter 10 should have preceded the chapter on prices of production (Chapter 9). Had this been done, then it would be clear that Marx's 5 capitals found on the left of the tables in Chapter 9 are based on market values, or representative values, and not on individual values.

The cold hard fact is that individual values cannot be transformed directly into prices of production. They can only be transformed into market values. Only market values or market prices can be transformed into prices of production. The first transformation of value into price takes the form of the emergence of market values around which market prices oscillate.

The second transformation, the one dealt with by Marx in Chapter 9 is the subject of this article. Before preceding to this subject, the third transformation, the one orchestrated by the industrial cycle needs to be briefly mentioned. Suffice to say, it is only over the entirety of the industrial cycle that total prices will equal total values. In the down-phase, less value will be realised because of depressed prices, and in the up-phase, value will be over realised by the elevation of prices. Thus, the viewing of total prices and values cannot be done frame by frame, but only over a sequence of frames (a video so to speak) extending over a number of years.

The transformation solution.

Marx's intention in Chapter 9 is two-fold - to illustrate how much surplus value needs to be redistributed, and, in what direction – so as to achieve a uniform rate of profit between capitals of differing composition. To keep his example simple, he assumes a consistent rate of exploitation of 100% so that each worker contributes the same amount of surplus value to the pool of surplus value.

Table 2 below reproduces the second table found in Chapter 9 (with the exception of the final row.)

Table 2.

Capitals	Surplus value	Value of commodities	Cost price of commodities	Price of commodities	Rate of profit	Divergence price vs value
(1)	(2)	(3)	(4)	(5)	(6)	(7)
i) 80c+20v	20	90	70	92	22%	2
ii) 70c + 30v	30	111	81	103	22%	-8
iii) 60c + 40v	40	131	91	113	22%	-18
iv) 85c + 15v	15	70	55	77	22%	7
v) 95c + 5v	5	20	15	37	22%	17
390c+110v=500	**110**	**422**	**312**	**422**	**22%**	**+26-26=0**

(page 256, Volume 3 Penguin Edition)

It takes a total of 26 to achieve a uniform rate of profit (column g), comprising 8 + 18 donated and 2, 7 and 17 received. The donors of this surplus value are capitals ii, and iii and the receivers are capitals i, iv and v. The direction in all cases of this redistribution is from the capitals of below average composition to the capitals of above average composition. The proof that this is a arithmetical example, is not to be found in the terminology used by Marx, where he avoids using the term "price of production" settling instead for "price of commodities" (column g), but in the fact that once the capitalists withdraw their 110 in profit, the capitals revert back to 100, there starting point.

This is illustrated in the table below.

Table 3.

Capitals	Price of commodity	Less profit withdrawn	Equals Cost Price	plus unconsumed capital	equals capital
(1)	(2)	(3)	(4)	(5)	(6)
I	92	22	70	30	100
Ii	103	22	81	19	100
Iii	113	22	91	9	100
Iv	77	22	55	45	100
v	37	22	15	85	100
Totals	**422**	**110**	**312**	**188**	**500**

At the end of the cycle of production, the value in existence is 610 comprising 500 capital and 110 profit (surplus value). The amount of money the capitalists receives upon sale is 422 in total (the sum of the prices of commodities). Their wealth at this point consists of 422 in sale receipts plus 188 of unconsumed c or means of production. When they withdraw their 22 each of profit from production in order to unproductively consume it, they have 312 left (422 – 110) which when added to their 188 of unconsumed capital restores there original 500 of capital.

Thus, the end point becomes the starting point and we can redo the example over and over again only to return to the exact same number. This proves that what is taking place is a simple arithmetical model to demonstrate two axioms and nothing more.

The question therefore arises; is it possible to go beyond this to provide a proof rather than an example, while preserving the following invariable: - total price must equal total value, and, total profit must equal total surplus value? The answer is yes. And when we do this, the real resolution of the transformation problem is found. To begin.

The total value in existence at the end of the production cycle preparatory to being sold amounts to 610 comprising 500 capital and 110 surplus value. What needs to be priced, the 610 or only the 500? The answer is the entire 610 not the 500 because in accordance with Marx's assumptions, and due to the use of market values, one element cannot be priced while the other is left at its original value. It would be methodologically incorrect to end up with a mishmash of reproduced and embodied prices.

There are two further assumptions. Firstly, that reproduction is simple reproduction, meaning that none of the 110 is converted into additional capital, and that the quantity of money advanced to absorb the output of each capital remains unchanged.

Step 1. Here lies the first novel element of this solution, the one hitherto hidden, but without which it is impossible to convert value into price. The total of 26 surplus value has to be split into two streams. One to reprice capital and the other to adjust profits so that capitals now differently priced still yields the same uniform rate of profit of 22%. The denominator in all cases is 610 representing the value of the social product which includes capital and profit. 110 profit divided by 610 equals 18% and 500 capital divided by 610 equals 82%. The division of the two streams is found in Table 4 below.

Table 4.

Capitals. (Chapter 9 notations)	**Redistribution of surplus value**	**Allocated to capital (82%)**	**Allocated to Profit (18%)**
(1)	(2)	(3)	(4)
i	+2	1.6	0.4
ii	-8	-6.6	-1.4
iii	-18	-14.6	-3.4
iv	+7	5.7	1.3
v	+17	13.9	3.1
	=26	**=21.2**	**=4.8***

(* Rounded off)

Step 2. In repricing capital only 21.2 of surplus value will be redistributed not 26, as in the case of Marx's price of commodities found in Table 3 above. The result will be capitals which differ from 100.

Table 5.

capital	**Capital at market value**	**Surplus value distributed**	**Repriced capital**
(1)	(2)	(3)	(4
i	100	1.6	101.6
ii	100	-6.6	93.4
iii	100	-14.6	85.4
iv	100	5.7	105.7
v	100	13.9	113.9
totals	**500**	**+21.2-21.2=0***	**500***

(* rounded off)

Immediately we are presented with a problem, the price appreciation and depreciation of capital is clearly the end of the process not the beginning. It requires making explicit, the mechanism for achieving this result.

Step 3. According to Marx's figures the value in circulation to be consumed consists of 422 (312 capital and 110 profit), while the unconsumed or dormant capital amounts to 188. It follows that only the 422 can enter into the repricing of capital, or more precisely the 312 of capital, because the 110 in profit is unproductively consumed as revenue and lost to reproduction. Only the 312 that is thrown back into reproduction can become the mechanism for this repricing. To determine by how much individual prices making up the 312 must shift to achieve this repricing of capital, the following procedure can be used. For ease of observations figures have been rounded up. (There is an additional proof in my first article on the transformation problem.)

Table 6.

Capital	**Newly priced capital**	**Less unconsumed capital**	**Equals replenishing cost price.**	***Original cost price at market value***
(1)	(2)	(3)	(4)	(5)
i	102	30	72	*70*
ii	93	19	74	*81*
iii	85	9	76	*91*
iv	106	45	61	*55*
v	114	85	29	*15*
	500	**188**	**312**	***312***

The cost price has to be sufficient, so that when the capital returns in its expanded or reduced monetary form, it is sufficient to appreciate or depreciate each capital back to its newly priced form. That takes place in the fourth column titled "replenishing cost price". For example, Capital v receives back 29 which when added to its unconsumed capital of 85, swells v's capital to 114 in monetary terms. Conversely in Capital iii, the replenishing cost price falls to only 76 from 91, resulting in iii's capital being reduced to 85 because 76 in cash is received rather than 91. These deviations are in order, because it is to be expected that the deviations will be most extreme in those capitals whose compositions deviate furthest from the average.

Step 4. Having put the price into cost price it is possible to proceed further to obtain the <u>initial</u> price of production or cost price plus the profit margin of 22%. To do so, the two streams of surplus value must be reunited reforming the original 26 in surplus value distributed. The remaining 4.8 will now be used to adjust the mass of profit in order to compensate for capitals no longer equally priced.

Table 7.

capitals	**Priced capital**	**Adjusted profit**	**Rate of Profit**	**+ Cost price**	**= Price of production**	***Price of commodities****
(1)	(2)	(3)	(4)	(5)	(6)	(7)
i	102	22.4 (+0.4)	22%	72	94.4	92
ii	93	20.5 (-1.5)	22%	74	94.5	103
iii	85	18.7 (-3.3)	22%	76	94.7	113
iv	106	23.3 (+1.3)	22%	61	84.3	77
v	114	25.1 (+3.1)	22%	29	54.1	37
totals	**500**	**110 (4.8)**	**22%**	**312**	**422**	**422**

(*taken from Table 2.)

In Table 7 the two streams of surplus value are reunited and now total 26. Of this 21.2 has repriced capital while the 4.8 has adjusted profits so that a uniform rate of profit prevails for each capital. In column (3) the additions or deletions from profits appear and they total 4.8 in both directions. Thus 18.7 over 85 yields 22% just as 25.1 over 114 also yields 22%. No longer are capitals all 100 and no longer are profits always 22. This completes the definition of prices of production which now reads: the market price of production is the price which yields an average rate of profit, not on the original capitals, but on the newly priced capitals.

The final proof that this solution is in order is found in Table 8 below. We note that after the capitalists have withdrawn their aliquot share of profit, all capitals remain at their new prices and do not revert back to their original form of 100. The two tests have been met, namely that the initial price of production must maintain the price of the capital that generates it and it must provide sufficient profit so as to yield a uniform rate of profit on that repriced capital.

Table 8.

capital	Prices of production	Less profit	= cost price	+ unconsumed capital	= Total capital
(1)	(2)	(3)	(4)	(5)	(6)
i	94.4	22.4	72	30	102
ii	94.5	20.5	74	19	93
iii	94.7	18.7	76	9	85
iv	84.3	23.3	61	45	106
v	54.1	25.1	29	85	114
Totals	**422**	**110**	**312**	**188**	**500**

We notice that the prices of production deviate from Marx's "prices of commodities" found in Chapter 9. (Table 7) Further that the deviations are at their greatest where the composition of capital rises furthest above, or falls furthest below the average, more specifically capitals iii and v. Finally, the spread in Marx's "Prices of Commodities" at 56%, is wider than the spread of 36% found in Table 8.

I would reject the argument and would contest it, that this solution disproves Marx. My resolution is more comprehensive. The solution presented here takes an additional two steps not found in Chapter 9, that is all. Both Marx and I are on the same methodological train, all that has happened is that Marx has alighted at an earlier stop while I have continued to the terminus.

Additional Step 5. It could be argued that an additional step is needed. Critics could point out that in Table 8 above, the unconsumed capital is still priced at its old value. Therefore, to head off any future criticism, and prevent the kind of criticism of Chapter 9 - the issue of incongruity - the final step is undertaken below. This step alters the aggregate cost price. This is due to the repricing of the unconsumed capital.

We recall that the value composition of capital in this series is 390/110. There is thus a preponderance of constant capital which is also concentrated in the capitals of above average composition where the bulk of unconsumed capital also resides. This is due to proportionately less c being consumed in i, iv and v. As these above average composition capitals are the beneficiaries of the redistribution of capital, there is a generalised rise in the price of their constant capital, which now exceeds the original value of the unconsumed capital.

To distinguish the prices found in Table 8 from Table 9, the former selling price is referred to as the initial price of production, and the latter as the market price of production.

Table 9.

original unconsumed c	Overall Prices (capital + profit)	Less profit = capital	Less adjusted c unconsumed	Equals cost price	plus profit	= Market Price of production.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
30	124.4=81.6c+20.4v+22.4p	102	30.6 (+0.6)	71.4	22.4	93.8 (-0.6)
19	113.5=65.1c+27.9v+20.5p	93	17.7 (-1.3)	75.3	20.5	95.8 (+1.3)
9	103.7=51c+34v+18.7p	85	7.7 (-1.3)	77.3	18.7	96.0 (+1.3)
45	129=90.1c+15.9v+23.3p	106	47.7 (+2.7)	58.3	23.3	81.6 (-2.7)
85	139.1=108.3c+5.7v+25.1p	114	96.9 (+11.9)	17.1	25.1	42.2 (-11.9)
188	**610=396.1c+103.9v+110p**	**500**	**200.6 (+12.6)**	**299.4**	**110**	**409.4 (-12.6)**

Column (2) represents the total price of capital plus profit equal to 610. The p denotes that surplus value has now been converted into the profit realised by each capital. When this profit is deducted it yields the newly priced capital (3). Column (4) is the repriced unconsumed capital using the same physical ratios Marx used in the tables found in Chapter 9. Take the first row. The original unconsumed capital was 30 out of 80. Column 2 shows that this 30 is now priced at 30.6 (81.6/80 x 30 = 30.6) or +0.6. Once this newly priced unconsumed capital is deducted from the total capital, the cost price is found in column (5). Finally, by adding the individual mass of profits to the new cost prices, the market prices of production are found (7).

Aggregate numbers do not vary. Total capital remains priced at 500 comprising 299.4 consumed and 200.6 unconsumed in contrast to 312 consumed + 188 unconsumed before. Corresponding to the increase in the price of these unconsumed stocks of constant capital, total sales will fall from 422 to 409.4. As this fall in sales represents reciprocal demand for goods, it is reasonable to assume this reduced demand will negate any price movements in the market resulting from the demand side.

The final point to make is that the prices of production in column 7 represent the fabled equilibrium prices so sought after in the market. They cannot change. They are mathematically derived, immutable, pure final prices.

To move on we have to depart the world of mathematics and enter the physical world where prices are driven by the restless movement of capital itself, rather than a calculator.

Step 6. To overcome the limitations of mathematics we need to enter the realm of physics. Prices change, not because accountants are at work, they change because of the physical movement of capital itself. All that the above maths tells us, is how much capital must be redistributed and in what direction. On the other hand, the movement of capital which alters production by reducing it in the donor capitals and increasing it in recipient capitals, also alters market prices by its movement.

To maintain our assumptions, we assume capital transits in a manner which retains the aggregate composition of capital at 390c + 110v. We also assume the monetary demand for each product remains unchanged so that any price effects result solely from changes in the volume of production due to the prior movement of capital. This is a viable assumption based on a closed system where the outputs from one capital form the inputs of the others.

Table 10 is the restatement of the original starting point. It assumes that the capitals of 100 each have been priced and are no longer at market value. The quest is to determine what market values correspond to these priced capitals with their unique profits and cost prices. The answer will reveal that these market values need to differ from the ones used by Marx in his Tables.

Table 10.

capitals	Capital priced at	Price of commodities	Less profit withdrawn	Equals cost price
(1)	(2)	(3)	(4)	(5)
i	100	92	22	70
ii	100	103	22	81
iii	100	113	22	91
iv	100	77	22	55
v	100	37	22	15
totals	**500**	**422**	**110**	**312**

Table 10, as has been said, assumes that values have previously been converted into prices which forms the starting point. Due to the peculiar fact that all capitals are uniformly priced at 100, we can dispense with two flows of surplus value and divert the full 26 to capital alone.

We know a priori, that capitals i, iii and iv were in receipt of surplus value amounting to 2, 7 and 17 (26) to price their capitals at 100. We also know that capitals ii and iii donated this 26 in lumps of 8 and 18. To work backwards, to move from prices to values these flows must now be reversed and what was lost, restored. This 26 must be restored in such a way that the aggregate c to v ratio is maintained at 78% and 22% or 390c and 110v. This is achieved in the Table 11 below. The Donor Capitals donate in proportion to their c to v ratios creating an average composition of 23.8c + 2.2v. In turn these average ratios are applied to the Receiver Capitals to allocate the 8 and 18 worth of capital. This is the only method which retains the overall composition of capital over the 5 capitals.

Table 11.

Original Receiver	loses	Marx's ratios	Composition of distribution
(1)	(2)	(3)	(4)
i	2	(80c:20v)	= -(1.6c + 0.4v)
iv	7	(85c:15v)	= -(6.0c + 1.0v)
v	17	(95c+5v)	= -(16.2c+0.8v)
	=26		**= -(23.8 + 2.2v) = -26**
Original Donor	**gains**	**Average ratios**	**Average Distribution**
ii	8	(23.8c + 2.2v)	+(7.3c + 0.7v)
iii	18	(23.8c + 2.2v)	+(16.5c + 1.5v)
	26	**= +(23.8c + 2.2v) =26**	**+(23.8c + 2.2v) =26**

In table 12 below, these distributions yield the new market value

Table 12.

capital	Original market value	Distribution of capital	Composition of distribution	= new market value of capital
(1)	(2)	(3)	(4)	(5)
i	80c + 20v = 100	-2	-1.6c - 0.4v	78.4c + 19.6v = 98
ii	70c + 30v = 100	+8	+7.3c + 0.7v	77.3c + 30.7v = 108
iii	60c + 40v = 100	+18	+16.5c + 1.5v	76.5c + 41.5v = 118
iv	85c + 15v = 100	-7	-6.0c - 1.0v	79.0c + 14.0v = 93
v	95c + 5v = 100	-17	-16.2c - 0.8v	78.8c + 4.2v = 83
	390c+110v =500	**0**	**+26-26 = 0**	**390c+110v = 500**

Table 12 column (5) confirms the restorations have conformed to our assumptions and that the aggregate composition of capital remains 390c to 110v. However, the individual compositions have changed in capital ii and iii due to the investment of higher composition capital. This will mean each industry's contribution to surplus value will be altered but without altering the aggregate figure of 110 which is the goal. There is an additional redistribution of 2.2 in surplus value.

The total value of sales in their monetary form remains at 422 composed of 312c plus 110p. This 422 must be spent on 422 of output comprising c + v +s. In this closed system, outputs form inputs, thus fixed amounts of money are spent in each industry. The social demand expressed in money remains unaltered. If prices are not adjusted, then some of the output will not be circulated. In some industries there will be unsold goods, and in other industries idle money. This will disrupt production. Thus, in capitals ii and iii where production has expanded from its previous level, prices must fall to absorb this production because of the monetary demand. The opposite occurs in capitals I, iv and v where production has contracted from its previous level. These price movements are shown in Table 13 below column (3) and they correspond to the actual distribution of surplus value. Thus, there is an actual movement of surplus value via the medium of the monetary equivalent, from the above composition capitals (ii & iii) to the below average capitals (i, iv and v), in accordance with Marx's assumptions.

Table 13.

capitals	New market value of capital	Money difference	Repriced Capital	Surplus value	Used up c
(1)	(2)	(3)	(4)	(5)	(6)
i	78.4c + 19.6v = 98	+2	80c + 20v = 100	19.6s	50
ii	77.3c + 30.7v = 108	-8	72c + 28v = 100	30.7s	52
iii	76.5c + 41.5v = 118	-18	65c + 35v = 100	41.5s	43
iv	79.0c + 14.0v = 93	+7	85c + 15v =100	14.0s	40
v	78.8c + 4.2v = 83	+17	95c +5v = 100	4.2s	10
totals	**390c+110v = 500**	**0**	**397c +103v =500**	**110**	**195**

The rate of exploitation remains at 100%. Therefore, the adjusted expenditure of labour found in column (2) represented by v gives rise to a similar amount of s found in column (5). In column (3) the money differential is applied back to the capitals to price capital in column (4). We note that due to the change in the composition of capital in ii and iii, c + v deviates from Marx's original figures here. (This is due to the above convention of using averages for the transfers to the two below average composition capitals.) As we are now dealing with the price of labour power, no longer its value, we may assume that in ii and iii the price of labour power has been driven below its value and conversely in i, iv and v it has risen above its value. Differing individual pay scales now appear. In the order of capitals, workers either enjoy rises or suffer losses amounting to: +2%, -9%, -16%, +7% and +19%. This is not a forced abstraction but corresponds to reality where workers in the most labour-intensive industries bear the lowest pay rates while workers in the most capital (constant) intensive industries enjoy the highest pay. (Marx deals with these fluctuations in wages in greater detail in Chapter 11.)

The more important point to consider is not the change in the internals, but the aggregate numbers and whether these have changed. Following Marx's method, the various internals are priced below in Table 14 in the same order and using the same headings as is found in Chapter 9. Capital remains at 500. After deducting the 195 consumed c (Table 13) from the total amounting to 397c, a sum of 202c is yielded which when added to the "price of commodities" amounting to 408 (methodology as per Chapter 9; 298 cost price + 110 profit)) the original figure of 610 is obtained. (In Chapter 9 that figure

was composed of 422 plus 188). Thus, all is in order. **Total Prices = Total Values & Total Profits = Total Surplus Value.**

The difference in internals merely respond to the changed physical conditions of production in ii and iii where production has expanded, compared to i, iv and v where production has contracted. In the tables below columns 2 + 4 + 5 harks back to the data found in Chapter 9 of Volume 3, but with this qualification, all capitals are now priced.

Table 14A.

capitals	Repriced Capital	Surplus value Table 13	Cost price of commodities (Chapter 9)	Profit (Chapter 9)	Price of Commodities (4) + 22 profit	*Cost price + surplus value (4) + (3)*
(1)	(2)	(3)	(4)	(5)	(6)	*(5)*
i	80c + 20v = 100	19.6s	70	22	92	*89.6*
ii	72c + 28v = 100	30.7s	80	22	102	*110.7*
iii	65c + 35v = 100	41.5s	78	22	100	*119.5*
iv	85c + 15v =100	14.0s	55	22	77	*69*
v	95v +5c = 100	4.2s	15	22	37	*19.2*
totals	**397c =103v =500**	**110**	**298**	**110**	**408**	***408***

The final point to note is that the redistribution of surplus value amounting to 26 no longer suffices to yield an average rate of profit. It now requires 28.2 as demonstrated in Table 14B below.

Table 14B.

capitals	(Table 14A) Cost price + surplus value	(Table 14A) Price of Commodities	Difference between columns 2 & 3	Rate of Profit	s.v. (table 13) plus difference column (4)
(1)	(2)	(3)	(4)	(5)	(6)
i	89.6	92	+2.4	22%	(19.6 + 2.4)/100 = 22%
ii	110.7	102	-8.7	22%	(30.7 – 8.7)/ 100 =22%
iii	119.5	100	-19.5	22%	(41.5-19.5)/100 =22%
iv	69	77	+8	22%	(14 + 8)/100 = 22%
v	19.2	37	17.8	22%	(4.2 + 17.8)/100 = 22%
totals	**408**	**408**	**+28.2 – 28.2 = 0**	**22%**	**(110 + 0)/500 = 22%**

And finally, as Table 14C below shows, the surplus value stream of 28.2 has been implicitly split into two streams once more, with 26 repricing capital and 2.2 providing the final adjustment to profits. As this 2.2 forms part of the 110 in profit revenue withdrawn from production, there is no effect on the repricing of capital which continues to stand at 100 in each case, nor does it affect the market value of output. Here again lies the importance, as Marx emphasised, of distinguishing revenue from capital.

Table 14C.

capitals	Difference between price and value (Table 14B)	Distribution to capital (Table 12)	Difference attributable to profits
(1)	(2)	(3)	(4)
i	+2.4	+2.0	+0.4
ii	-8.7	-8.0	-0.7
iii	-19.5	-18.0	-1.5
iv	+8.0	+7.0	+1.0
v	17.8	+17.0	+0.8
totals	**+28.2 – 28.2 = 0**	**+26 – 26 = 0**	**+2.2 – 2.2 = 0**

Far from contradicting Marx the above alterations prove his axioms. Earlier, when capital was redistributed in Table 11, 2.2v was transferred to ii and iii boosting their production of surplus value by 2.2s. In turn this reduced the production of s in capitals i iv and v. Thus, an additional 2.2s had to be distributed from ii and iii to complete the equalisation of the rate of profit.

Thus the following proof could also be added. It is clear that the loss of, or addition of money, will not only affect the pricing of capital but profit as well once capital is priced. Here we apply the effect of the changes in monetary income to profits themselves plus the inclusion of the revenue adjustment of 2.2 due to the change in the conditions of production.

Table 14D. (Additional proof)*

capitals	Repriced Capital	Money difference	Surplus value	s.v. + or - new money	Plus 2.2 adjustment
(1)	(2)	(3)	(5)	(6)	(7)
i	80c + 20v = 100	+2	19.6s	19.6 + 2 = 21.6	21.6 + 0.4 =22
ii	72c + 28v = 100	-8	30.7s	30.7 – 8 = 22.7	22.7 -0.7 = 22
iii	65c + 35v = 100	-18	41.5s	41.5- 18 = 23.5	23.5 -1.5 = 22
iv	85c + 15v =100	+7	14.0s	14.0 + 7.0 = 21.0	21.0 + 1.0 = 22
v	95c +5v = 100	+17	4.2s	4.2 + 17 = 21.2	21.2 + 0.8 = 22
totals	**397c +103v =500**	**0**	**110**	**110**	**+2.2 – 2.2 = 0**

* Of course it is possible to avoid the 2.2 if a lesser amount of capital was transferred in the first place and not the 26.

Table 15 finally brings the two associations together which were developed above. To the left of the red line are the capitals in their market value form found in Chapter 9 and the repriced capitals which would be associated with this particular market value. On the right of the red line we find the only market value which can give rise to capitals priced at 100.

Table 15.

Capitals	Marx's original Value Capital	Repriced capitals		Priced capital	Capital measured by corresponding value
(1)	(2)	(3)		(4)	(5)
I	80c+20v=100	102		100	78.4c + 19.6v = 98
ii	70c+30v=100	93		100	77.3c + 30.7v = 108
iii	60c+40v=100	85		100	76.5c + 41.5v = 118
iv	85c+15v=100	106		100	79.0c + 14.0v = 93
v	95c+5v=100	114		100	78.8c + 4.2v = 83
	390c+110v =500	**500**		**500**	**390c+110v = 500**

The directions in both cases between values and prices are the same. In the case of capital iii, a market value of 100 gives rise to a smaller priced capital of 85 in the first instance, while a market value of 118 gives rise to a smaller capital of 100 in the second instance. Conversely, capital v with a market value of 100 gives rise to a larger priced capital of 114, while, with a market value of 83, it gives rise to a priced capital of 100.

The difference is due to their different capital compositions alone. Capital iii is below average composition, while v is an above composition. The specific price movement found in above average composition capitals, namely that prices always rise above values, is of the highest historical significance. It explains why the movement of investment goes from horse drawn carts to locomotives rather than from locomotives to horse drawn carts, which would be the case if low composition capitals did not yield up some of their surplus value to high composition capitals via the pricing system.

FROM SIMPLE TO EXPANDED REPRODUCTION.

The methodologies used above can be applied just as well to expanded reproduction. Once again, the procedure begins with Marx's original market values and value of commodities.

Table 16.

Capitals	Market Value	Surplus value	Value of circulating commodities	Cost price
(1)	**(2)**	**(3)**	**(4)**	**(5)**
i	80c+20v=100	20	90	70
ii	70c + 30v=100	30	111	81
iii	60c + 40v=100	40	131	91
iv	85c + 15v=100	15	70	55
v	95c + 5v=100	5	20	15
	390c+110v=500	**110**	**422**	**312**

The changed assumptions are as follows. Capital I and iii originally provided luxury goods which were unproductively consumed as profit revenue, therefore not thrown back into production. The recipients of some of these profits now decide to accumulate rather than consume, and they decide to invest 20 back into i and 20 back into iii. In other words, 40 out of the 110 of profit is no longer unproductively consumed but is about to be productively invested.

To examine the effect of this change on production we jump over the "ideal" or transitional stage where investment increases capital from 500 to 540 (in accordance with Marx's method in Volume 2) but without as yet altering output, to the stage of expanded (altered) production. This change is detailed in Table 17 below.

Table 17.

Capitals	Market Value	Surplus value	Value of commodities	Cost price	Unconsumed capital	Rate of profit
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**
i	96c+24v=120	24	108	84	36	20%
ii	70c + 30v=100	30	111	81	19	30%
iii	72c + 48v=120	48	157	109	11	40%
iv	85c + 15v=100	15	70	55	45	15%
v	95c + 5v=100	5	20	15	85	5%
Totals	**418c+122v=540**	**122**	**466**	**344**	**196**	**23%***

(*rounded off)

As a result of the increase in capital and the resulting increase in production, the social product has increased from 610 to 662. Within that total, the amount of capital has increased from 500 to 540, surplus value from 110 to 122. Additionally, the value of circulating commodities has increased from 422 to 466. The rate of aggregate surplus value has risen to 22.6% or 23% rounded up, as against 22% originally.

Table 18.

Capitals	Market Value	sv	Sv redistributed	Rate of profit (rounded off)	81.6% sv to capital	18.4% sv to profit

(1)	(2)	(3)	(4)	(5)	(6)	(7)
i	96c+24v=120	24	+3	27/120=23%	2.4	0.6
ii	70c + 30v=100	30	-7	23/100 =23%	-5.7	-1.3
iii	72c + 48v=120	48	-21	27/120=23%	-17.1	-3.9
iv	85c + 15v=100	15	+7.5	22.5/100=23%	6.1	1.4
v	95c + 5v=100	5	+17.5	22.5/100=23%	14.3	3.2
Totals	**418c+122v=540**	**122**	**28-28=0**	**122/540=23%**	**22.8-22.8**	**5.2-5.2**

The steps applied here correspond to the steps applied earlier in the article. The redistribution of surplus value needed to establish a uniform rate of profit is detailed in column (4) of Table 18. This time, because of the larger social product, 28 rather than 26 has to be redistributed. Again, because of the different rate of surplus value, 81.6% of that 28 needs to be diverted to reprice capital (6), and 18.4% to adjust profits (7), to achieve a uniform rate of profit based on the repriced capital. The results of column (6) and (7) are then applied to columns (2) and (5) in Table 19 below.

Table 19.

Capitals	Repriced capital	Less unconsumed capital	Equals New cost price	Plus adjusted profit	Rate of profit	Price of production
(1)	(2)	(3)	(4)	(5)	(6)	(7)
i	122.4	36	86.4	(27+.6)=27.6	23%	114
ii	94.3	19	75.3	(23-1.3)=21.5	23%	96.8
iii	102.9	10.8	92.1	(27-3.9)=23.1	23%	115.2
iv	106.1	45	61.1	(22.5+1.4)=23.9	23%	85
v	114.3	85	29.3	(22.5+3.2)=25.8	23%	55.1
Totals	**540**	**196**	**344***	**122**	**23%**	**466***

(*rounded off)

Once again it can be argued that the unconsumed capital has not been repriced. This is done in Table 20 below which yields the Market Price of Production, in other words, unless conditions change, these equilibrium prices are immutable.

Table 20.

Capitals	Repriced capital	Less unconsumed capital	Equals New cost price	Plus adjusted profit	Rate of profit	Market Price of production
(1)	(2)	(3)	(4)	(5)	(6)	(7)
i	122.4	37 (+1)	85.4	(27+.6)=27.6	23%	113 (-1)
ii	94.3	17.9 (-1.1)	76.4	(23-1.3)=21.5	23%	97.9 (+1.1)
iii	102.9	9.3 (-1.5)	93.6	(27-3.9)=23.1	23%	116.7 (+1.7)
iv	106.1	47.7 (+2.7)	58.4	(22.5+1.4)=23.9	23%	82.3
v	114.3	97 (+12.0)	17.3	(22.5+3.2)=25.8	23%	43.1
Totals	**540**	**209 (+13)**	**331**	**122**	**23%**	**453 (-13)**

The difference in the initial price of production needed to reprice capital and adjust profits, as against the market price of production, is 13. Market prices of production have been priced down by 13, because the unconsumed capital has been priced up by 13.

Conclusion.

Why did Marx style Chapter 9 as a problem rather than a solution? The answer is that capitalist pricing is an ever-present problem, because the evolved prices of commodities do not represent their essence, or what is the same thing, their values, or what is the same thing, their actual costs of production, or what is the same thing, their weighted average labour times. The deviation of prices from values expresses the contradiction found at the heart of the capitalist mode of production, namely that while commodities are products of labour these products circulate as products of capital at prices which reward profits not labour. This dynamic contradiction is only resolved in a socialist society based on the abolition of capital, where prices now reward labour directly and therefore represent actual costs of production.

The transformation problem therefore seeks to explain the continuous interaction between value and price in a world dominated by profit and shaped by the movement of capital. It is for this reason I consider Marx to have been a double-sided physicalist and who, in contrast to the deviations between price and value, would never have deviated from this dialectical standpoint. Had Marx lived until 1988 he would not be of changed mind, only extremely old. (My criticism of TSSI or the sequentialists can be found on the following link https://theplanningmotivedotcom.files.wordpress.com/2020/07/tssi-pdf.pdf)

Prices move and will always move in a capitalist society within the gravity well created by value, but always in an erratic orbit influenced by the incessant motion of capital. This article remains true to the inner connection between market value and price and how changes in one explain changes to the other because in the end prices have to respond to changes to actual costs of production or what is the same thing changes to market values. It therefore remains loyal to Marx's methodology despite the fact that the market prices of production arrived at here, differ from the prices of commodities found in Chapter 9.

But Chapter 9 is more than a recognition that prices deviate from values. This mere recognition marks only the starting point for a scientific enquiry which must also address whether these deviations are random and without margin, or whether they have a specific direction and a definite limit. In short, whether they are regulated or not despite this regulation being chaotic. By explaining what sets the direction and the weight of movement, therefore the mode of regulation, Marx provided the essential methodology making possible the difficult modelling of these deviations.

This is what this article has employed in a novel way. It demonstrates how one set of market values relates to another set of market prices of production. That a definite and strong association exists between them. That the one is explained by the other. That changes to one corresponds to mediated changes to the other beginning with changes to cost-prices. In so doing, it establishes the law of value as the final arbiter of prices.

This article has proven that prices can be derived from market values, and in turn market values can be derived from prices. The latter being vital, because any starting point for modelling the real world, has to begin with the actual circulating prices which obscure market values. The article further demonstrates that the methodology used for simple reproduction is transferable to expanded reproduction. Therefore, the assumptions and methodology described here provides the maths and formulae needed to model the entire economy. Whether this is desirable or doable is another matter. More importantly, by demonstrating that there are no longer any incongruities between price and value, this article negates the criticisms of the law of value that has dogged Chapter 9 for well over a century.

Finally, we observe that it is this regulated deviation of price from value which ensures that the elastic pricing system found within capitalism, needed to reward profit rather than labour, does not end up over-stretching and systemically disrupting capitalist production itself. Capitalism may appear to be chaotic, but there is order to this chaos, and the laws that order the capitalist mode of production, are the laws first identified by Karl Marx.

Brian Green, June 2020.

Postscript.

This article supersedes the four previous articles on the transformation problem by being more complete and methodologically precise. However, this article could not have been written, had I not written the earlier articles in which I developed the methodologies which are refined and expanded here.