SOLVING FOR THE NUMBER OF SALES USING THE CORRELATION BETWEEN GROSS OUTPUT AND GROSS VALUE IN THE NATIONAL ACCOUNTS.

The Bureau of Economic Analysis, hereafter referred to as the BEA, is the USA equivalent of the British Office of National Statistics (ONS). This paper provides a more detailed analysis of the formula first used to prepare a more accurate rate of profit by converting annual wages to variable capital.

In preparing the National Accounts, the BEA prepares two series - gross output (GO) and gross value (GV). Gross output is the total price of all the sales in a given year which is a mixture of intermediate sales and final sales. Final sales are defined as those sales which result in a product being consumed rather than being worked up or sold on. For example, if water is drunk from a bottle, it is a final product, if it is used to manufacture squash it is an intermediate one. Before its consumption or final sale, the emerging product may change hands many times.

To avoid double counting, the BEA subtracts intermediate sales from total sales or GO. The result is a figure comparable to final sales. The value of final sales turns out to be equal to Value Added or GV. Finally once GV is determined, it is used to prepare the National Accounts including crucially, national income. All will be made clear as we proceed.

The difference between the GV and GO figure is elegantly explained by the BEA in one of its pamphlets, obtainable on the web and entitled *Measuring the Economy: A Primer on GDP and the National Income and Product Accounts* (2014: pages 3 and 4). We will use its examples to explain the problem and the solution. A farmer grows wheat, the first intermediate sale, and sells it to a miller to convert into flour, the second intermediate sale, who then sells it to the baker where the third and final sale occurs when the bread is sold to a customer who eats it. There are thus two intermediate sales and one final sale, a total of three sales.

Assuming for the sake of convenience that all three producers add £10 of value to their product then; the farmer sells the wheat to the miller for £10. The miller sells the flour to the baker for £20 made up of £10 value added and £10 for the cost of the wheat. The baker finally sells the bread for £30, £10 value added and £20 for the cost of the flour.

The final sale of £30 is equal to the £10 value added by the farmer, the miller and finally the baker and so gross value added comes to £30 (excluding wear and tear for the time being). But total sales come to £60 or double the value added by the three. The farmer sells the wheat for £10, the miller sells the wheat for £20 (the miller's £10 plus the farmer's £10) and the baker sells the bread for £30. Gross output is twice as large as value added, which is gross output less intermediate sales. Visually this is shown in the table below. We note that the value of the final sale, that of bread, is equal to the total of value added of 30 or GV (as mention earlier). This is always the case.

	Sale number	Value added	Value of Sale		Intermediate or final?
	1	10	10	=10	Intermediate (farmer)
	2	10	10+10	=20	Intermediate (miller)
	3	10	10+10+10	=30	final (baker)
TOTALS		**30 (GV)**		**60 (GO)**	

What we are trying to solve for are the number of turnovers that links GV to GO. In our example they are given, but when, later we turn to national statistics they are not. All we are given are (a) GO, (b) GV and (c) intermediate sales. We know there are only a finite number of sales that can reconcile GV with GO. Outside this range it is impossible to maintain the GV amount while retaining the GO amount.

In our example the possible range is from 2 to 3. It would be possible to have only two sales or turnovers but would it yield 30 (GV) and 60 (GO) as before. The miller could buy out the farmer or vice versa so that the sales of 10+20 = 30 is reduced to only one sale of 20. In this case GV remains at 30 but GO is reduced to 50 violating our rule that both remain as before. So two sales are ruled out.

Two sales resulted in a GO below 60. Conversely if we use 4 sales as we have done below, GO always rises above 60. So we know the number of sales must be greater than two and less than four.

5	5	4	4
10	15	7	11
5	20	9	20
10	30	10	30
30	**70**	**30**	**65**

Accordingly we have developed the following formula to solve for turnover times:

$$\frac{GO}{GV} + \frac{(GO-GV)}{GV}$$

And when we apply the figures from our above example we find that it yields

$$\frac{60}{30} + \frac{(60-30)}{30} = 2 + 1 = 3.$$

This is the correct answer. If we were to add in a fourth sale, where for example the baker makes pie cases to sell on instead of bread, then we would have GV of 40 and GO of 100 or 10+10+10+10 versus 10+20+30+40. Our calculation would thus be;

$$\frac{100}{40} + \frac{(100 - 40)}{60} \text{ or } 2.5 + 1.5 = 4$$

Again the formula provides the correct answer. The problem emerges when we examine national statistics where we are given only GV (value of final sales) GO and intermediate sales (GO-GV). When we look at GO for the National Economy in the USA we find the GO figure for 2013 was $26291.6 billion and the figure for GV was $14556.4 billion. In other words GO was approximately 180% of GV or 1.8 times larger.

We are not provided with the number of turnovers for 2013. We know it cannot be less than two.

G.V.	G.O.
0.8	0.8
0.2	1.0
1.0	1.8

We know it is improbable that there are only two sales and that the entirety of the intermediate sales will comprise a single turnover. And we know it cannot be as high as three. For try as we may three turnovers always yields a result where GO exceeds 1.8 as shown below.

Example 1		Example 2		Example 3	
0.3	(=0.3)	0.4	(=.4)	0.1	(=.1)
0.5	(.5+.3=.8)	0.4	(.4+.4=.8)	0.7	(.1+.7=.8)
0.2	(.3+.5+.2=1)	0.2	(.4+.4+.2=1)	0.2	(.1+.7+.2=10)
1.0	**2.1**	**1.0**	**2.2**	**1.0**	**1.9**

Accordingly we know the range of probability is greater than two but less than three. And when we apply our formula we arrive at 2.6, which is within that range.

$$\frac{1.8}{1.0} + \frac{(1.8-1.0)}{1.0} = 1.8 + 0.8 = 2.6$$

The formula suggests 2.6 is the most probable number of sales for the US economy consistent with gross value at $14556.4 billion and gross output at $14556.4 billion, or to put it more simply the Value added each year of $1 is made up of the value added by 2.6 sales or turnovers.

Next let us turn to manufacturing where the available figures stretch from 1977 through to 2013. If we take the figures for 2013 we find gross value at $2028.5 billion and gross output at $5940.3 billion. We notice straight away that the difference between the two figures are much bigger at 2.9 rather than 1.8 for the whole economy. This means that intermediate sales this time exceeds final sales, they are a 190% bigger rather than being 20% smaller when looking at the whole economy. From this we can deduce that there are more duplicated sales, therefore more turnovers, and indeed our formula shows that this to the case.

$$\frac{5940.3}{2028.5} + \frac{(5940.3 - 2028.5)}{2028.5} = 2.93 + 1.93 = 4.86 \text{ (or 4.9 rounded off)}$$

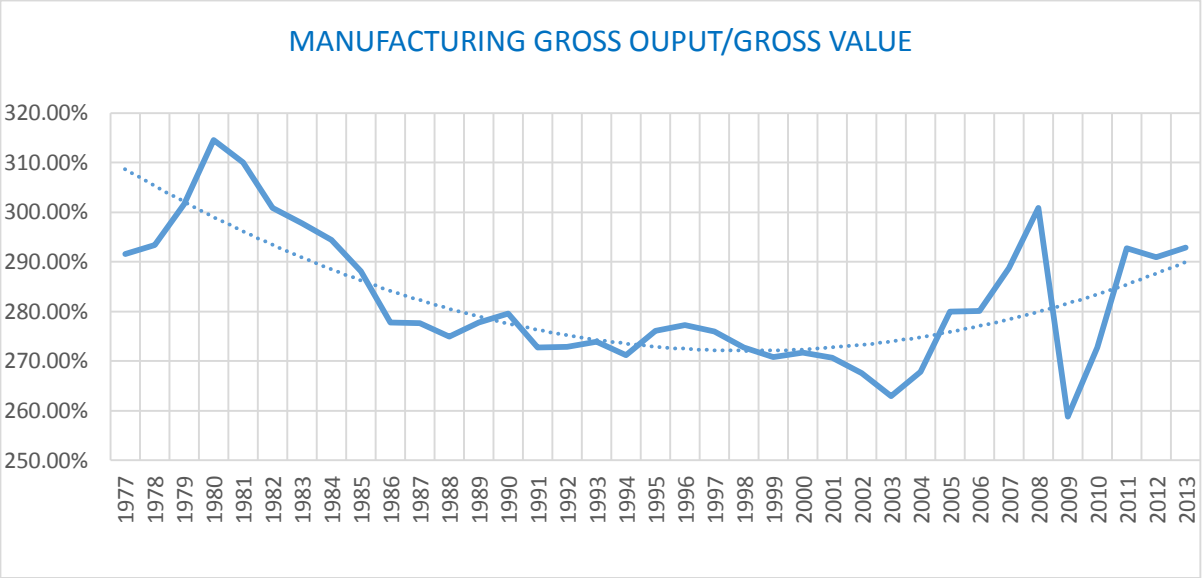
In other words the number of turnovers in manufacturing amounts to 4.9 per annum. In the table below we have reproduced the series from its inception in 1977.

Table 1 US Manufacturing Gross Output/Gross Value and Turnover 1977-2013

Year	**1977**	**1978**	**1979**	**1980**	**1981**	**1982**	**1983**	**1984**
GO/GV	2.9	2.9	3.0	3.1	3.1	3.0	3.0	2.9
Turnover	4.8	4.9	5	5.3	5.2	5	5	4.9
Year	**1985**	**1986**	**1987**	**1988**	**1989**	**1990**	**1991**	**1992**
GO/GV	2.9	2.8	2.8	2.7	2.8	2.8	2.7	2.7
Turnover	4.8	4.6	4.6	4.5	4.6	4.6	4.5	4.5
Year	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**
GO/GV	2.7	2.7	2.8	2.8	2.8	2.7	2.7	2.7
Turnover	4.5	4.4	4.5	4.5	4.5	4.5	4.4	4.4
Year	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**
GO/GV	2.7	2.7	2.6	2.7	2.8	2.8	2.9	3.0
Turnover	4.4	4.4	4.3	4.4	4.6	4.6	4.8	5.0
Year	**2009**	**2010**	**2011**	**2012**	**2013**			
GO/GV	2.6	2.8	2.9	2.9	2.9			
Turnover	4.2	4.5	4.9	4.8	4.9			

Source: BEA Interactive tables: Industry GDP 2015

The series shows that turnover fluctuates in a range of 25% between 4.2 in 2009 and 5.3 in 1980. It is responsive to periods of expansion and periods of contraction, though the recession in 1980 seems to be the exception. We will return to this observation later when we deal with foreign outsourcing in the 1980s. There is a long term slowing down in the number of turnovers beginning in the 1980s which bottoms out in 2003. After that it increases progressively only to fall back spectacularly in 2009 when the financial crisis paralysed industry for a brief period, after which it bounced back.



These turnovers apply to a calendar year. For the economy as a whole this is 2.6 turnovers and for manufacturing it is roughly 4.9. If we convert this to periods measured in days, we have an average turnover period of nearly 140 days for the economy (365 days divided by 2.6 turnovers) and for manufacturing we have an average turnover period of 73 days.

A turnover period in the Marxian sense is the period between the two sets of exchanges that constitutes the capitalist social relation or more specifically the circuit of capital. The first is the purchase of the factors of production necessary for production. The second and concluding exchange is the sale of the products produced in the interim. Marx described it thus

M – C … P … C' – M'

We start with money **M or money capital**. Money buys the factors of production as commodities **C**. There is a gap …as production is organised and commences. **P** then takes place…and at the end of the production period there are new commodities **C'** which are then sold for **M'**. The ' denotes more money is received than is paid out. In other words the circuit of capital yields a surplus.

Now it is important to remember that the circuit of capital is not limited to the production period (**P**) but to the period between **M** going out and **M'** coming in. This is the most common confusion. For example Car Companies boast about being able to assemble a car in under a day. But that is only the final stage of building a car, which does not include building the engine, stamping and pressing the parts, painting, sub-assembling, assembling, testing and correcting faults, storage preparatory to delivery and that second sale. And it does not include the 3-5 years taken to design, test and bring into

production the new car. The claim that it takes only hours to produce a car is a fiction, one which will be revealed in our table on page 7, where we look more closely at Ford and General Motors.

We can now return to the manufacturing industry. When we talk of gross value and gross output we are describing billions of individual sales that occur with the space of a calendar year. Corporations do not have to finance these sales for a year but only for the period until sale when new money comes in. Our formula revealed that the average turnover period for manufacturing is 73 days based on 5 rounds of turnover per annum. 73 days is the average period between the two exchanges.

Accordingly when we talk of turnover periods we are not describing individual sales. In the opening section we used the simple model of a farmer, miller and baker. We now have to put this out of mind as it could be misleading. What we are now dealing with is aggregated sales, the value of countless millions of individual sales that occur within a given period. It is the period we are now interested in and will concentrate on.

If we return to the manufacturing industry, the turnover period amounts to 73 days (10.4 weeks if you like). Five turnover periods yields a calendar year of 52 weeks. We can thus describe the value added in 2013 in the following way (seasonally adjusted).

	G.O.	**G.V**
Jan 1-March 12	$405.7bn	$1188.1bn
March 13 – May 25	$405.7bn	$1188.1bn
May 26 – August 4	$405.7bn	$1188.1bn
August 5 – October 18	$405.7bn	$1188.1bn
October 19 – December 31	$405.7bn	$1188.1bn

Together the columns add up to the annual totals for Gross Value and Gross Output. On the basis of seasonal adjustments, benign inflation, I.V.A. and C.C.A. adjustments by the BEA we may assume that the value added in each period is similar, though never identical, but for our purposes the variation would be insignificant. Our formula yields the periods and the consistency of period is the corollary to the formula. This point about the similarity between the periods within such a short time frame, in this case a calendar year, needs to be emphasised. The formula works because the value in each period does not deviate significantly from the next one because of the various adjustments undertaken by the BEA. As a result each period can be considered average for the year.

Otherwise the formula could break down. For example if we deliberately underweighted the early periods and then overweighted the later periods it would be possible to create 6 turnovers for manufacturing and even 7 possibly. But that would be a mathematical fiction rather than what is happening in the real world. In the real world the value produced in each turnover period is similar after seasonal adjustments. In the case of manufacturing each period, as we have seen above, would be $413.9 billion for value added $1212.3 billion for sales.

The real test of course is to see if the result yielded by the formula conforms to actual results found in manufacturing. As we will see in the table below, some industries have a shorter average period and some industries have a longer period. We resort to this table as verification of the formula. This is an examination of about twenty of the largest corporations in manufacturing occupying the major specialities.

Bearing in mind the formula M-C…P…C'-M' we intend to measure turnover periods in three different ways covering the whole circuit or part of it. The first is using working capital:

$$\frac{\textbf{Cost of annual sales}}{\textbf{Working capital}} \times \frac{1}{365} = \textbf{number of days}$$

As we shall see, this is the closest formula to Marx's and Engels' formula for the circuit of capital. Cost of sales or cost of production is the annual amount spent on wages for production workers. To this is added the annual purchases of raw materials, components, ancillary materials, power needed for production. In turn this is adjusted by the materials and finished goods etc. (inventory for short) unsold and brought into this year from the previous financial year less the inventory unsold at the end of this financial year which will be carried forward to the following year.

Unfortunately the cost of sales is understated by many companies who fail to add Research and Development to the cost of annual sales. R&D has been removed in order to be added to the balance sheet as a fixed asset which is a nonsense. It is worth commenting on the treatment of overheads. The wages paid to non-productive staff called SG&A [Selling, General and Administrative expenses] is deducted from profits and does not appear as a component of cost of production. In this the capitalists mimic Marx's distinction about labour expended in production and labour expended in circulation, labour that produces profits and labour that is a deducted from profits.

Working capital is the actual amount of circulating capital a corporation needs to finance current production and is composed of current assets less current liabilities. *'…the whole capital cannot be simultaneously employed in production. One part of this capital therefore always lies fallow, whether in the form of money capital, stocks of raw materials, finished but still unsold commodity capital, or outstanding debts that are not yet due for payment*.' Capital Volume 3, Penguin Edition page 163. (We may assume that included in the phrase 'outstanding debts' are bills payable and bills receivable, credit from suppliers and credit to customers.) There are problems with working capital not anticipated by Marx and Engels and that is the one year rule. Any asset or liability with a maturity of less than one year is lumped into working capital regardless of whether this capital is connected to production, to the purchase of inputs or the sale of outputs. Ideally working capital should comprise cash plus inventory plus bills receivable (credit advanced to customers) less bills payable (credit received from suppliers). This can be isolated by examining individual balance sheets, but as we are merely seeking to prove an equation this amount of detail is unnecessary.

The proof of the importance of working capital is its use in determining whether a firm is solvent or going insolvent. If working capital is growing then the company is profitable because current assets are growing compared to current liabilities and if it is contracting or goes negative then the company could be making losses (unless the current assets have been used to acquire additional fixed or longer term assets).

The second definition is operating cycle. This covers the period from production to sale or **P…C'-M'** and comprises the amount of days' worth of unsold inventory together with the number of days a firm has to wait to collect its money for any product it has sold on credit. Most products, except in retail are sold on credit rather than cash (even in retail credit is extended in the form of credit cards, store cards and personal loans). Operating cycle is useful because it reveals the minimum amount of capital to cover current production and the period between sale and cash received. The formula is:

Days of inventory + days between sale and receipt of money = operating cycle.

The third definition is the inventory cycle, or how many times a year it takes for inventory to turn over. This can then be recalculated in days as we have shown above. The quicker the inventory turnover a corporation achieves within a given industry, the fewer days of stock it holds, the more efficient it is deemed to be. The inventory cycle is limited to **…P…** and while it is only a segment of the circuit of capital is often confused with the complete circuit. For example in the car industry as we have already discussed, the production period and the circuit of capital are two different things. The formula is:

cost of annual sales x 1 = **number of days**
inventory **365**

We now reproduce these three turnover periods below. These figures are obtained from *Stock Analysis on Net: Top 100 Leaders.* We begin with segments of manufacturing before proceeding to examine individual corporations. All figures in days. Listings are in order of operating cycle. We recall that our original formula yielded a turnover period of 73 days.

INDUSTRY	WORKING CAPITAL		OPERATING CYCLE	INVENTORY CYCLE.
Industrial Sector	43	(Note 1)	138	87
Chemicals	82		114	70
Basic materials	83		108	76
Pharmaceuticals	135		105	61
Personal Products	n/a	(Note 2)	103	63
Consumer Goods	22		76	48
Technology	119		67	26
Cars	13		61	36
Computer Hardware	46		56	21
General Electric	n/a		118	79
Caterpillar	77		166	112
ABB	85		179	76
Eli Lilly	18		263	203
Merck	126		178	122
Dow Chemicals	79		91	62
Du Pont	96		181	132
3M	66		131	44
Monsanto	105		226	48
Boeing	45		254	223
United Technologies	38		139	76
General Motors	43		58	36
Ford	n/a		39	23
Kimberly Clark	n/a		103	53
Colgate Palmolive	19		106	70
Procter & Gamble	n/a		86	58
Nike	114		139	94
Apple	10		42	53
H.P.	21		73	13
IBM	39		53	165
Intel	77		106	77
Microsoft	287		118	36

(**Note 1**. R&D is not included as it is now considered to be fixed capital and not part of working capital. This has shortened the operating and inventory cycles in industries where R&D is a significant expense

for example the pharmaceutical and car industries. In the car industry we would need to add about 8 days to each ratio to accommodate the level of R&D.

Note 2. n/a denotes that current liabilities exceed current assets so working capital is negative)

From this sample of 21 large corporations, amounting to close to 40% of manufacturing gross output, we may conclude that our previously used formula yields a good approximation of the number of turnover periods found in manufacturing. Five sectors sit above it, one is average and two which happen to be the largest components of manufacturing, sit below it. We are of course referring to the 5 turnovers our formula yielded equivalent to 73 days. A simple average for working capital yields 68 days (a weighted average is more desirable) which is within 7% of the 73 days yielded by our formula.

The inventory cycle too is important. If it exceeded the period of 73 days it would invalidate the formula, but with a simple average (a weighted average again is required) of 54 days, it is twenty five percent less than the 73 day period, thereby inhabiting the zone suggested by the formula. Finally turning to the operating cycle, we would expect it to exceed 73 days which it does (92 days) as it includes bills receivable but excludes bills payable which would reduce it. So in every case the three cycles confirms the yields anticipated by the formula.

Leontief and Kutznets, those disavowed Marxists, who nonetheless employed Marx's method developed in Volume 2 of Capital to eliminate duplication, have contributed to the science behind the understanding and interpretation of the capitalist economy and it's dynamic. As for the academic Marxists, most of whom religiously regurgitate Marx and Engels writings, arguing over this interpretation and that, where the quest for purity is all consuming, the less said the better. Marxism is not a theoretical science, it is an applied science, one that needs to investigate today's world, and even if we get it wrong on occasion, or our research reveals shortcomings, so what, it brings us closer to understanding the real world and its movement. The body of data out there is deep and extensive and cries out to be investigated. Marx would have rejoiced in the availability of this data.

That is why the author will not dwell on Chapter 4 of Volume 3 of Das Kapital, mainly edited by Engels. Engels insight into his world occurred a 140 years ago. He may have found the annual number of turnovers for the cotton industry to be 8.5, but that has to be qualified by his own admissions, unreliable source material and unknown working capital or circulating c + v. Those who have criticised the result of 5 turnovers for manufacturing over the recent period have done so on the bases that if turnovers were so high already 140 years ago, they should be much higher now because of the phenomenal advance in technology both in the sphere of production and in circulation.

This one sided interpretation of history ignores counter-vailing factors, and we know, economic development is never simple but complex, it consists of a dominant trend offset by factors that modify its trajectory. Here we list some:

1. The increased size of corporations (concentration and centralisation of wealth) meaning that what was once produced by many independent producers is now produced by fewer companies. Value added is now added in bigger lumps.
2. More complex products are produced which take longer to produce. Boeing is an outstanding example where 223 days of inventory are held. A jet plane takes longer to produce than a steam engine, one of the most complex pieces of equipment in Engels' day.
3. Production has become truly global. Raw materials may be mined in one continent, transported to another for turning into components which are then shipped to another country for turning into subcomponents then shipped to yet another country for final assembly – generally China – before finally being shipped to all the other countries for sale. Most of this is sea borne and a container typically takes 30 days from China to Europe or 20 days to the East Coast of the USA or 30

days to the Gulf or West Coast. To this must be added 2-3 days on either side for loading and clearance. Hence a great deal of inventory at any time is en route.

4. Just-in-time. This reorganisation of production, both a function of technology and management skills has speeded up the inventory cycle by reducing the number of days of inventory held and to a degree has offset the dispersion of production.

5. Other outsourcing offsets. 4000 suppliers typically supply each car company whose productive activity is reduced to assembly and engine construction. This reduces the length of their turnover period, as demonstrated in the tables above for Ford and GM. This acts against the concentration and centralisation of capital in many instances but again, because this outsourcing is international and involves a great deal of travel, turnover times for the industry are not as reduced as would be expected from just in time and outsourcing. What is rarely considered is this. Corporations will accept a slower turnover time, if the cost reductions exceed the cost of capital. Now that wages are rising in China for example, so that the absolute gap in wages is not so stark, corporations are beginning to relocate production closer to their markets, often returning production to their own country, as in the case of Apple and General Electric. If this process accelerates we can expect a reduction in turnover time.

6. Research and Development is now a larger share of the expenditure of labour time. To produce products may take less time, but the time needed to develop and test a new car for example can take three to five years. The amortisation of R&D across the entire production period for the product can add 10% and more to the turnover time when adjusted for days.

The ability to distil turnover times from the National Accounts has eluded most Marxist until now. The connection between Gross Value and Gross Output has not been made. Whether or not Leontiev was aware it yielded turnover times cannot be known, but it was always implicit in the methodology he developed.

The importance of developing turnover times cannot be underestimated. On the one hand it enables Marxists to develop a more accurate rate of profit by converting annual wages into variable capital. On the other it helps to determine the potential profitability of production. Two capitals of equal size employing equal numbers of productive workers who suffer the same rate of exploitation will yield two different rates of profit measured annually if one has a shorter turnover time than the other.

In 2008, when circulation was paralysed due to the collapse of Lehman Brothers, turnover times captured by the graph above, elongated dramatically. It was one of the primary reasons for the collapse in profitability during this time. Once circulation regained momentum, profits snapped back as turnover times contracted sharply. At the beginning of all recessions, the breakdown in circulation which paralyses production, increases turnover times deflating profits.

This paper was written to explain in greater detail and to substantiate the formula;

$$\frac{GO}{GV} + \frac{(GO-GV)}{GV}$$

It has succeeded in its intent. We are now able to more accurately represent profitability in all its forms with recourse to the national accounts.

Brian Green. March 2015